

Find ${}_{300}C_3$. You must show intermediate steps, so do not use the ${}_nC_r$ button on your calculator.

SCORE: _____ / 4 PTS

$$\boxed{\frac{300!}{3! 297!}} = \frac{\overset{50}{300} \cdot 299 \cdot 298 \cdot 297!}{6 \cdot 297!} = \underline{4,455,100}$$

Prove that $n! > 2^n$ for all integers $n \geq 4$ by mathematical induction.

SCORE: ____ / 12 PTS

PROOF:

BASIS STEP: $4! = 24$

$$2^4 = 16$$

$$4! > 2^4$$

INDUCTIVE STEP: ASSUME $k! > 2^k$ FOR SOME PARTICULAR BUT
ARBITRARY INTEGER $k \geq 4$

[PROVE $(k+1)! > 2^{k+1}$]

$$(k+1)! = (k+1)k! > (k+1)2^k > 2 \cdot 2^k = 2^{k+1}$$

SINCE $k \geq 4$

$k+1 \geq 5$

SO $k+1 > 2$

GRADED

BY

ME

BY MI, $n! > 2^n$ FOR ALL INTEGERS $n \geq 4$

Find the coefficient of $x^4 y^5$ in the expansion of $(2x - 5y)^9$.

SCORE: ____ / 5 PTS

$$\text{GENERAL TERM} = \binom{9}{r} (2x)^{9-r} (-5y)^r \rightarrow r=5$$

$$\begin{aligned} \binom{9}{5} (2x)^4 (-5y)^5 &= \overset{\textcircled{1}}{126} (16x^4) (-3125y^5) \\ &= \overset{\textcircled{1}}{6,300,000} x^4 y^5 \end{aligned}$$

Use the Binomial Theorem to expand and simplify $(3x - 4y)^5$.

SCORE: ____ / 7 PTS

$$1(3x)^5(-4y)^0 + \underbrace{5}_{\textcircled{\frac{1}{2}}} (3x)^4(-4y)^1 + \underbrace{10}_{\textcircled{\frac{1}{2}}} (3x)^3(-4y)^2 + \underbrace{10}_{\textcircled{\frac{1}{2}}} (3x)^2(-4y)^3 + \underbrace{5}_{\textcircled{\frac{1}{2}}} (3x)^1(-4y)^4 + 1(3x)^0(-4y)^5$$

$$= \underbrace{243}_{\textcircled{\frac{1}{2}}} x^5 - \underbrace{1620}_{\textcircled{1}} x^4 y + \underbrace{4320}_{\textcircled{1}} x^3 y^2 - \underbrace{5760}_{\textcircled{1}} x^2 y^3 + \underbrace{3840}_{\textcircled{1}} x y^4 - \underbrace{1024}_{\textcircled{\frac{1}{2}}} y^5$$

Write the repeating decimal $0.31\overline{8}$ as a simplified fraction. **NOTE: Only the 18 is repeated.**

SCORE: _____ / 7 PTS

$$0.3 + 0.018 + 0.00018 + 0.0000018$$

$$= \boxed{\frac{\overset{\textcircled{1}}{3}}{10}} + \boxed{\frac{\frac{18}{1000}}{1 - \frac{1}{100}}} \textcircled{2}$$

$$= \frac{3}{10} + \frac{\frac{18}{1000}}{\frac{99}{100}}$$

$$= \frac{3}{10} + \frac{18}{1000} \cdot \frac{100}{99} = \frac{3}{10} + \frac{1}{55} = \frac{33+2}{110} = \frac{35}{110} = \frac{7}{22}$$